Phys 410
Spring 2013
Lecture #2 Summary
25 January, 2013

Mass is a measure of an object's inertia, or resistance to acceleration. It is also the 'gravitational charge' of an object. This coincidence is known as the Principle of Equivalence. Mass is measured in kilograms (kg).

A force is an influence that produces acceleration of an object. Its direction is the direction of the resulting acceleration. A force of 1 Newton will produce an acceleration of 1 m/s^2 of a 1 kg mass.

We discussed Newton's Laws of motion for point-like particles. The first law states that an object that is subjected to zero net force will move with constant velocity. The second law says that the net force acting on the object will always equal the mass times the acceleration of the object. The third law says that forces always occur in pairs.

For a point-like particle that does not change its mass while in motion, the second law can be stated in terms of the linear momentum of the particle, $\vec{p} = m\vec{v}$ as $\vec{F}_{net} = \dot{\vec{p}}$, where the "dot" denotes derivative with respect to time.

The first law is actually contained as a special case of the second law. So why spell it out explicitly? The answer is that the first law helps us to distinguish frames of reference that are 'inertial' from those that are 'non-inertial.' An inertial reference frame is one in which Newton's first law holds true. If we prepare a particle such that it has zero net force and observe that it moves with constant velocity, then we know that we are in an inertial reference frame. As a counter-example, we considered the rotating (accelerating) frame of reference on the surface of a rotating table (while on the rotating table you experience centripetal acceleration). An observer in this frame would witness an object subjected to zero net force undergo acceleration, i.e. a change in the direction of its velocity vector as a function of time. This was illustrated with a friction-less ice cube moving on the surface. By the way, we will later 'patch up' Newton's laws of motion to work in non-inertial reference frames by adding new forces to account for the acceleration.

A consequence of Newton's third law of motion is the possibility of conserving the total momentum of a collection of particles interacting with each other with arbitrary forces. If the net external force on a set of N interacting particles is zero, the total momentum of that system, $\vec{P} = \sum_{\propto=1}^{N} \vec{p}_{\propto}$, is conserved. As an example, consider two particles colliding, but subject to no net external force. If they collide elastically, then both energy and momentum of the two-particle system are conserved. If they collide perfectly inelastically (i.e. they collide and stick together) then mechanical energy is not conserved, but the total momentum IS conserved. This robust conservation law is very useful in many branches of physics.

Finally we considered Newton's second law of motion in two-dimensional polar coordinates (r, φ): $m\ddot{\vec{r}} = F_{net}$. The basic issue is that the polar unit vectors $\hat{r}$ and $\hat{\varphi}$ change direction as the particle moves about the plane. Note that $\hat{r}$ is defined as the direction of increasing r coordinate at fixed φ coordinate, and similarly for $\hat{\varphi}$. Both unit vectors are functions of the angular coordinate φ. They can be written in terms of the directionally-invariant Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ as $\hat{r} = cos\varphi\, \hat{\imath} + sin\varphi \hat{\jmath}$, and $\hat{\varphi} = -sin\varphi\, \hat{\imath} + cos\varphi \hat{\jmath}$. A moving particle is parameterized by the time-dependent functions $r(t)$ and $\varphi(t)$, yielding a time-varying pair of unit vectors $d\hat{r}/dt = \dot{\varphi}\, \hat{\varphi}$ and $\frac{d\hat{\varphi}}{dt} = -\dot{\varphi}\, \hat{r}$. Starting from the coordinate vector $\vec{r} = r\hat{r}$, one can take two derivatives to find the rather complicated result $\vec{a} = (\ddot{r} - r\dot{\varphi}^2)\hat{r} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{\varphi}$.